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V.N.Strel'tsov

THE ELECTRIC FIELD OF A CURRENT-CARRYING CONDUCTOR

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A serias of papers [1-4] devoted to the problem of charge of a current-carrying wire has been recently published. This problem comes really to the subject of electrical field round such a conductor. For this the problem of relativistic invariance of the Gauss's law [3] was touched upon in the light of the approach expounded in the known Berkeley physics course [5] "Electricity and Magnetism" by Purcell. As one affirms there, according to special relativity theory for a space integral in the Gauss's theorem, we have

$$\int_{A^*(t^*)} \vec{E}^* d\vec{s}^* = \int_{A(t)} \vec{E} d\vec{s}. \qquad (1)$$

Here, for example, the left side represents the rest S^*-system (proper system of the conductor); $\vec{E}^*$, the electric field; and $d\vec{s}^*$, an element of the closed area A^*. The quantities in the right-hand side concern a moving S-system.

In fact, the corresponding relativistic invariant expression takes the form

$$\int F_*^{ik} ds_{ik}^* = \int F^{ik} ds_{ik} . \qquad (2)$$

Here F^{ik} is the electromagnetic field tensor; ds_{ik}, an element of the antisymmetrical 4-tensor of an area

$$ds_{ik} = -\epsilon_{ik\ell m} dx^\ell \delta x^m , \qquad (3)$$

where $i,k... = 0,1,2,3$, $\epsilon_{ik\ell m}$ is the Levi-Civity symbol ($\epsilon_{0123} = -1$). Habital quantities $d\vec{s}$ are given by time components $ds_{0\alpha}$ (α, β = 1, 2, 3), whereas $ds_{\alpha\beta}$ represent the projections of an area element on three "time planes" $x^0 x^\alpha$ ($x^0 = ct$) in Minkowski space, i.e. they depend on dx^0 and δx^0.

If the elements of area A^* are taken simultaneously (dx_*^0, $\delta x_*^0 = 0$ and hence $ds_{\alpha\beta}^* = 0$) in accordance with the radar formulation of relativity theory (see, e.g., [6,7]) in the S^*-system, the left side of (2) comes to the left side of (1). However, in the S-system with necessity already $ds_{\alpha\beta} \neq 0$, therefore the right side of (2) will come to the right side of (1) only in

that case if the components describing magnetic field $F^{\alpha\beta} = \vec{H} = 0$; in other words, if there are no moving charges creating the field in the S-system. On the basis of expressions (3) and Lorentz transformations, transformation formulae for ds_{ik} can be obtained. For simplicity we assume that the S*-system moves along the x-axis of the S-system with velocity v_x, and the directions of their axes coincide. Taking into account equation $ds^*_{\alpha\beta} = 0$, we find

$$ds_{01} = ds_x = ds^*_x, \quad ds_{02} = ds_y = ds^*_y \gamma, \quad ds_{03} = ds_z = ds^*_z \gamma, \tag{4}$$

$$ds_{12} = -\beta_1 ds^*_y \gamma, \quad ds_{13} = -\beta_1 ds^*_z \gamma, \quad ds_{23} = 0, \tag{5}$$

where γ is the Lorentz factor and $\beta_1 = v_x/c$.

On the other hand, as is known, the quantity $ds_{0\alpha}$ serves for the definition of such an important characteristic as the surface density of charge $\sigma^{0\alpha} = \vec{\sigma}$. For this the total charge of an area A is

$$Q = \int_A \vec{\sigma} d\vec{s} = \int_A \sigma^{0\alpha} ds_{0\alpha} . \tag{6}$$

Taking into account the other components ds_{ik} for relativistic generalization (6), we obtain

$$Q = \int \sigma^{ik} ds_{ik} . \tag{7}$$

Here $\sigma^{\alpha\beta}$ is the density of surface current, for example, σ^{12} is equal to the amount of electricity flowing for 1s through a unit of the surface section (i.e. the line) normal to the current direction. Similarly the known formula is generalized

$$\vec{E} = 4\pi\vec{\sigma} \quad \text{or} \quad F^{0\alpha} = 4\pi\sigma^{0\alpha} \tag{8}$$

For a relativistic invariant expression we have

$$F^{ik} = 4\pi\sigma^{ik} \tag{9}$$

and purely space components express obviously a relation between the density of surface current and the magnetic field. On the basis of (7) and (9) it is seen that in the relativistic case formula (6) is really not already total charge.

It should be noted that the practical statement of the problem considered in the mentional papers, strictly speaking, does

not answer the relativistic treatment of the Gauss's law. In fact, the discussed question comes to the following. Does the electric field appear around a closed electrically neutral conductor after current excitation in it? This means that the current appears only due to setting conductivity electrons in motion. Since for this the number of electrons does not change and as before it is equal to the number of positive ions, in our opinion the reply to the raised question can be received from consideration of the following simple example [8]. Let a pair of electric charges of different sings be and one of them (for example, negative) moves and the other is at rest. We begin with the simplest case when the distances between charges can be neglected. It is evident that the electric field of the (positive) charge at rest is given by the Coulomb potential

$$\phi_+ = -\frac{e}{R}, \tag{10}$$

where e is the electron charge. The field of a moving (with velocity $\vec{v}$) negative charge is given by Lienard-Wiechert's potential

$$\phi_- = \frac{e}{R - \vec{\beta}\cdot\vec{R}} . \tag{11}$$

where $\vec{\beta} = \vec{v}/c$. Thus, for summary electric fields we obtain

$$\phi = \phi_+ + \phi_- = \frac{e\vec{\beta}\vec{R}}{R - \vec{\beta}\vec{R}} . \tag{12}$$

As one can see, for $\beta = 0$ the field is really equal to zero. However, in the relativistic case the picture changes substantially. As is seen, the given couple is percieved by a trial charge as a neutral one only at angle $\pi/2$ to the direction of electron motion. For this it looks negatively charged in the "forward" direction and positively charged in the "back" directions as illustrated in the figure. It is evident that we have a great number of electrons and ions and, so to say, a "close" electron can be always found for a given ion at this instant. Indeed, the distance

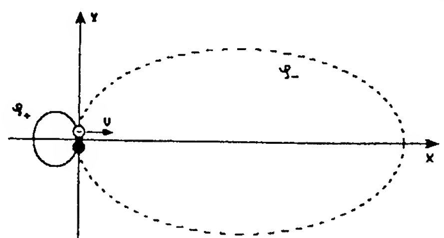

Fig. Electric equipotential of a charge pair, $v_- = 0.75$ c.

between neighbouring particles in the conductor is given by the microscopic value, whereas the distance R to the point of observation is an especially macroscopic quantity in this problem. Therefore the "nearest condition" used above is fulfilled indeed with a great accuracy.

The distribution of the electric field E can be calculated in an analogous way. The formula corresponding to (12) takes the form

$$\vec{E} = \frac{e}{R^3}\left[\frac{\gamma^{-2}}{(1-\vec{\beta}\vec{R}/R)^3}(\vec{R} - \vec{\beta}R) - \vec{R}\right]. \tag{13}$$

It is evident that in this case the picture of the field is more complicated, and its disappearance is related to the validity of the equality

$$\vec{R}[1-(1-\vec{\beta}\vec{R}/R)^3\gamma^2] - \vec{\beta}R = 0. \tag{14}$$

Thus, this consideration has shown that the appearance of electric field around the current-carrying conductor by no means testify to the change of its charge.

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Стрельцов В.Н. Д2-91-499

Электрическое поле проводника с током

Затронут вопрос релятивистской инвариантности теоремы Гаусса. Показано, что появление электрического поля вокруг нейтрального проводника после возбуждения в нем тока (без внешнего подвода электронов) не означает изменения его заряда.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1991

Strel´tsov V.N. D2-91-499

The Electric Field of a Current-Carrying Conductor

A subject concerning the relativistic invariance of the Gauss theorem has been discussed. The appearance of the electric field around the neutral conductor after excitation of current in it (without external admission of electrons) doesn´t signify the change of its charge.

The investigation has been performed at the Laboratory of High Energies, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1991